SUGGESTIONS FOR MODIFICATIONS IN THE ORDER AND TIME OF PRESENTING ALGEBRA

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This subject was suggested by the recent radical changes effected in the order of presenting arithmetic. A certain grammar-school arithmetic published in 1895 and the majority of arithmetics published before that time were based upon the false idea that each portion of the subject must be completely mastered before proceeding to the next. All of notation and numeration is given first, and on the eleventh page of this certain arithmetic the pupil is asked to read the number 1,555,676,410.62. Then follows all of addition, subtraction, multiplication, and division.

Since that time a number of arithmetics have appeared arranged on a spiral plan. In these newer publications simple work in the four fundamental operations is given first, then work of slightly increased difficulty, until finally the pupil is led by gradual stages to the more difficult portions of the subject. This plan has everywhere been accepted as being more psychologically correct than the old method of attempted complete mastery of successive topics.

But although this important change for the better has been effected in the order of presenting arithmetic, no change has been made in the order of presenting algebra. The futile attempt is still made to attain a complete mastery of the four fundamental operations as each is taken up and pupils are forced to become familiar with the most difficult cases of factoring before they are permitted to apply the simpler cases in the solution of easy quadratics. For instance, the following example is given in one of the elementary algebras, before any use of the simpler cases of factoring is made in the addition of fractions or in the solution of quadratics: "Factor m^2-n^2+2n-1." If the principle of complete mastery of successive topics is wrong in arithmetic, then it is wrong in algebra.

Before suggesting the changes that might be made in the order of presenting algebra, it would be well to determine the grade of the high or grammar school in which the subject should be introduced. In the great majority of schools algebra is not studied until the first or second years of the high-school course. But there are numerous reasons why its study should be commenced in the seventh or eighth grades, and a number of courses of study, including that of New York city, already require this.

There are not many practical applications of algebra outside of arithmetic, unless the pupil continues his work in college, but many of the examples of percentage, the indirect cases of interest, and proportion can be solved more easily algebraically than by the methods of arithmetic. This is proved by the necessity of examiners to state at the beginning of an arithmetic paper that examples are not to be solved algebraically. The best explanations of square and cube root are afforded by algebra.

The great majority of pupils leave school after completing the work of the eighth grade. Whatever of algebra they are to know must be taught these pupils, then, in the grammar school.

As subjects are now arranged, there is a wide gap between the grammar and the high school. The methods and subject-matter of the two schools are totally different, although the principle that education should be an even, continuous development is well established. If something of algebra is taught in the grammar school, one portion of this gap, at least, will be bridged.

It is also a common complaint, and undoubtedly true, that there is not enough material for difficult thought-work in the course of study for the latter years of the elementary school. This was recognized by the British commission, which recently investigated our schools, as one of the most glaring faults in our educational system. This evil could in part be remedied by the introduction of a grammar-school algebra.

All this argument tends to show that the study of algebra should be begun in the seventh grade of the elementary school. Arithmetic should be practically finished in the seventh grade, and algebra should constitute the principal part of the mathematics of the eighth grade.

Before suggesting the order in which this algebra might be pre-

sented, consider the changes that should be made in the present subject-matter of arithmetic in order that this introduction of algebra into the elementary school may be possible.

Many prominent educators are claiming that arithmetic has not the disciplinary value heretofore attributed to it, and are demanding that it be given a less prominent place in the curriculum. On the other hand, Mr. Goss, late superintendent of Indianapolis, believes that "the American public school especially from the fourth to the close of the eighth year of school life, presents, *aside from mathematics*, no body of science or language which, when mastered, is worth possession as knowledge, or in the attainment of which there is acquired a subtle skill or the elements of a generous culture." But it is doubtless true that much of the latter portion of the arithmetic has been bequeathed to us by the mediæval age and is of little or no value in these modern times.

Professor David Eugene Smith, in the *Teachers College Record* for March, 1903, after considerable argument, concludes that the child's interest in the quantitative side of life should be the highest immediate aim of the teacher of mathematics in the grades, just as his interest in the spiritual side is the highest immediate aim of the teacher of literature; and the nature of the child, together with the needs of society, should constitute the main standard in selecting subject-matter. If this is a true conclusion, then much of that which we now teach in arithmetic will finally go.

Many of the problems of denominate numbers have no practical application in the life of today. Obsolete tables and those peculiar to a single line of business or to a single profession, and examples in which more than three different denominations are involved, have no place in our arithmetic. In everyday life distances are never given in miles, rods, yards, feet, and inches, but in one or two denominations, as 5 yards, $2\frac{1}{3}$ feet, or $3\frac{1}{4}$ miles. The housewife or groceryman never finds it necessary to subtract 3 bushels, 2 pecks, 1 quart, 1 pint from 4 bushels, 1 peck, 5 quarts, 1 pint. Is it not entirely wrong to make pupils for several weeks find the actual time of places when in after-life they will invariably use standard time? Profit and loss is not worthy of a special chapter, for the problems are the ordinary ones of simple percentage. The indirect cases of inter-

est are of little value, and scarcely any time need be given to them. Only the elements of partial payments, annual interest, and compound interest are essential. Many of the examples of exchange, and stocks and bonds, are never met with in the business life of today, and the teacher will find that interest will be aroused, and that the pupil will get a better grasp of the realities of these subjects, if the financial page of the newspaper be used instead of the textbook. Because of the false idea of business life which they involve, many of the problems of commission and brokerage, and of insurance, should be omitted. Few of the newer arithmetics give the cumbersome process of compound proportion, and it might be well if simple proportion were left for the algebraic treatment. Square root and cube root should also be left for algebra.

After this weeding-out process has been completed, arithmetic can be greatly enriched by the introduction of many series of examples which would bring the pupils in touch with different phases of society, and with the phenomenal achievements which are being made in the United States along commercial and industrial lines. Is it not true that almost all the impressions, whether correct or incorrect, that we have of many lines of business and occupations were received from some example or series of examples in our arithmetic? All that many people know of Wall Street are the first impressions they received in the subject of "Stocks and Bonds." Then we can see how important it is that these impressions should be correct, and that pupils should become acquainted with as many phases of life as possible. How much more sympathy we should have with the newsboy if we had solved a series of examples giving the cost and selling price of his papers, and the results showing his daily and weekly sales and profits.

To give another illustration: Running through the northern part of New Jersey, and connecting Newark Bay with Easton on the Delaware River, is an old and picturesque canal. The railroads parallel it now and carry most of the freight that used to go by boat, but it was once an important factor in the transportation of coal from the Pennsylvania mines to New York. The boats are drawn by mules or horses, and the numerous locks and plains so retard their progress that eight days are required for the round trip. A

boat carries about 69 tons of coal, and the boatman receives \$0.47 a ton for transporting. During one round trip he feeds his mules 4 bags of oats at \$1.30 a bag, 200 pounds of feed at \$1.50 a 100, 1 bushel of corn at \$0.75, and long hay every night at \$0.25. For \$2 a week he employs a helper to drive the mules and assist with the steering. What is his average daily wage? Before the railroads were constructed he received \$0.84 a ton and he could carry 75 tons. What percentage of his present daily wage is that of forty years ago? This is only one of numerous series of examples that could be given to make our arithmetic full of interest, and which would bring the pupil into sympathy with many of the different walks of life about him.

But after these additions and subtractions have been made, the subject-matter of arithmetic will be much less than before, and this will give us the opportunity to introduce algebra into the seventh and eighth grades of the grammar school.

In what order shall we present this algebra? A spiral plan, of course, must be followed, but there is great danger of carrying this scheme to an absurd extreme. The successive loops of the spiral may be made to follow each other in such close succession that the pupil is completely bewildered and a sort of mathematical nausea results. For instance, in a certain arithmetic every first, eleventh, twenty-first, thirty-first, forty-first page is devoted to addition; every second, twelfth, twenty-second, thirty-second, forty-second page to subtraction; every third, thirteenth, twenty-third, thirty-third, forty-third page to multiplication; and so on. Such an arrangement cannot fail to do pupils more harm than the old method of complete mastery of successive topics, but we must all agree that the idea of the spiral system is the correct one if applied in a sensible manner. When pupils reach the seventh grade their minds are more mature than when the beginnings of arithmetic were made, and they are ready to think more deeply and to grasp more difficult ideas, so that there is even greater danger here than in arithmetic of making a too frequent turn of the spiral.

Begin the algebra during the first half of the seventh grade, and teach it in connection with arithmetic. The mathematics of the eighth grade might be entirely algebra, but in the applied problems

of both these grades pupils should be given the choice of solving by the algebraic or arithmetic methods.

The algebra of the seventh grade might consist of simple work in the four fundamental operations, the easier cases of factoring, linear and quadratic equations. This will be the first turn of the spiral. In the eighth grade review the work of the seventh grade, but go more deeply into the theory of the subject, and increase the difficulty of the problems. In addition to this, give something of the theory of exponents and coefficients, the binomial theory with positive, integral exponents, and square root. In the first year of the high school teach thoroughly, from the four fundamental operations to the theory of equations, some standard text. This is the third and last turn of our spiral, and the pupil should now be prepared to delve deeply into the subject and to get a firm grasp of its underlying principles, so that he will be able to make it his useful servant in all subsequent work in physics, chemistry, astronomy, and the higher mathematics.